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# Modelling Mobility Data during COVID-19 in Portugal with R-INLA

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# Introduction

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- **Human mobility was severely disrupted during the COVID-19 pandemic**, as restrictions and behavioural shifts led to **unprecedented movement patterns** [1].
- Mobility data can act as a **proxy for interpersonal contact and transmission risk**, supporting the design of **adaptive mitigation measures**. Yet, behavioural adaptations may **weaken this proxy relationship** [2].
- Regression analyses indicate that **mobility effectively predicted weekly COVID-19 infection rates during the first wave** [3], and that **reduced mobility during the Omicron surge lowered peak incidence** [4].
- Integrating mobility data into epidemic models—particularly in early pandemic stages—**improved predictive accuracy** and **enhanced variant tracking**, outperforming models that ignored travel or used lagged indicators [5].

# Objectives

**Aim:** To model district-level mobility data from the **COVID-19 Google Community Mobility Reports** for Portugal [6].

## Specific objectives:

1. **Describe** mobility patterns and the **movement stringency** dynamics during the COVID-19 pandemic in Portugal.
2. **Build** a comprehensive dataset with **key predictors** explaining mobility variability.
3. **Develop and evaluate** time series models to accurately capture mobility trends.
4. **Identify** significant predictors and **characterize their effects** across mobility categories and districts.
5. **Assess** the impact of **stringency measures** on observed mobility.

Data

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### Google's COVID-19 Community Mobility Reports (CMR) [6]

CMRs are derived from **Google Location Services** (e.g., Google Maps). Although the data remain publicly available, **updates ceased in October 2022**. Reports cover multiple countries — including **Portugal** — and span from **February 15, 2020 to October 15, 2022**, across different spatial levels of detail.

- Data represent the **percentage change in mobility** relative to a pre-pandemic baseline, across six location categories:
  - Retail and Recreation
  - Grocery and Pharmacy
  - Parks
  - Transit Stations
  - Workplaces
  - Residential
- The **baseline** is defined as the **median mobility value for each weekday** during the period **January 3–February 6, 2020**, when mobility restrictions were not yet in place.

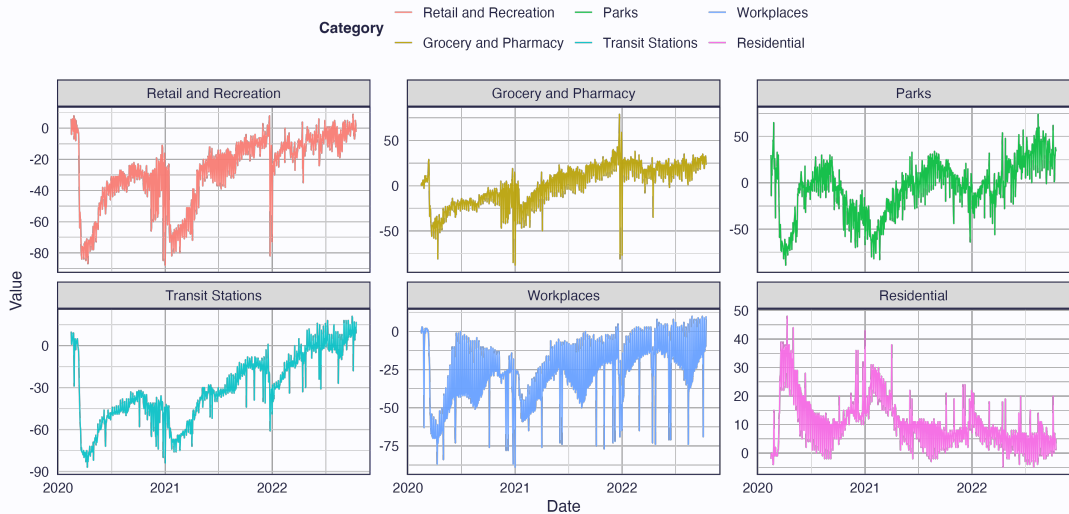


Figure 1: Overall mobility for mainland Portugal for all six categories.

During the COVID-19 pandemic, **Portugal and other countries** imposed movement restrictions, which were adjusted based on the **evolving situation**.

### Movement Stringency Index [7]

The University of Oxford developed the **Stringency Index**, a **numerical score** reflecting the level of public health measures and movement restrictions over time.

It is based on nine metrics: **school and workplace closures**, **public event cancellations**, **gathering restrictions**, transport closures, stay-at-home orders, public information campaigns, internal movement restrictions, and international travel controls.

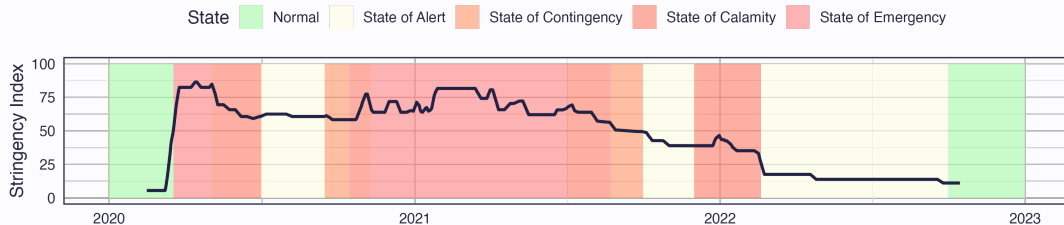


Figure 2: Movement stringency index overlaid on the Exceptional Legal Regime in force coded by colour.

### Climate Data Store (CDS) [8]

The CDS, operated by the **Copernicus Climate Change Service (C3S)**, provides free and open access to high-quality climate datasets — including **daily gridded temperature observations** across Europe.

- Higher temperatures have been associated with a **lower incidence of COVID-19** [9].
- Temperature variations may also **influence mobility patterns** over time, serving as an important explanatory variable.

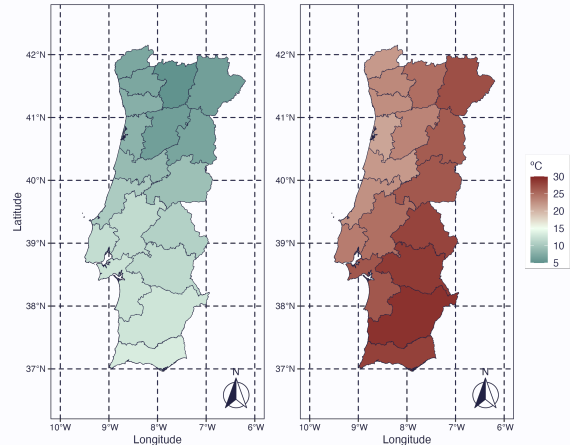
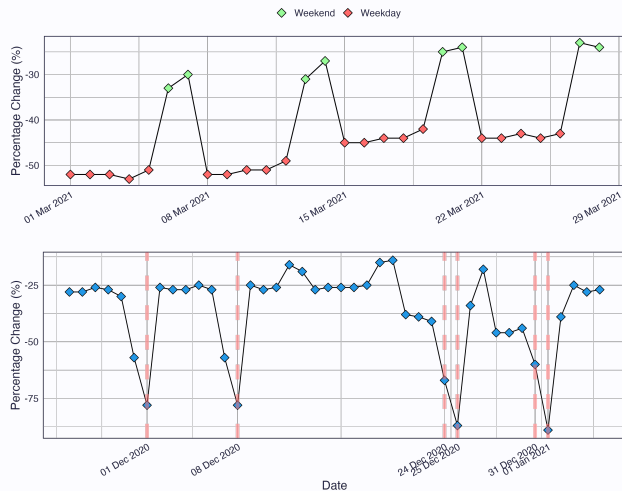
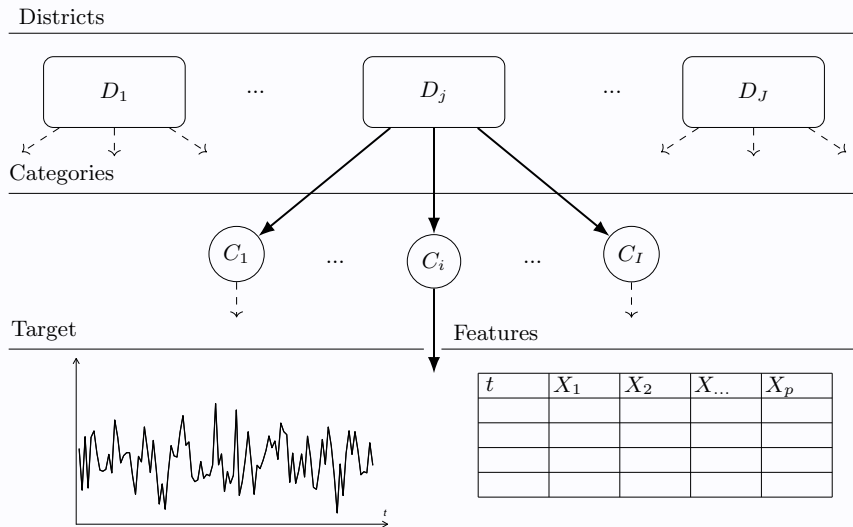


Figure 3: Mean temperature values on February 18th 2020 and August 15th 2021 per District.

- Mobility is measured as percentage change from a weekday-specific baseline.
- Clear **weekly patterns** emerge, particularly in:
  - Workplaces
  - Transit Stations
  - Retail & Recreation
- **National holidays** cause notable deviations in mobility, especially during weekdays.



**Figure 4:** Time series windows for percentage change in the Workplaces mobility category.



**Figure 5:** Schematic representation of the hierarchical dataset structure. For each District–Category pair  $(D_j, C_i)$ , where  $j = 1, \dots, J$  and  $i = 1, \dots, l$ , there is an associated distinct target time series and a set of features  $X_1, \dots, X_p$ , which may vary across time, space, or both. The representation is the same for each District–Category represented by the dashed lines.

Model

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**Hierarchical models** describe data with multiple levels, e.g., individuals within groups, spatial or temporal data [10].

**INLA (Integrated Nested Laplace Approximation)** is a fast alternative for **latent Gaussian models**:

- Approximates posterior distributions for parameters and random effects
- Efficient for large or complex hierarchical models
- Useful for random effects and spatio-temporal data

**Hierarchical model structure [11]:**

$$\mathbf{y} \mid \mathbf{x}, \boldsymbol{\theta} \sim \pi(y_i \mid x_i, \boldsymbol{\theta}) \quad (i = 1, \dots, n), \quad \mathbf{x} \mid \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{\mu}(\boldsymbol{\theta}), \mathbf{Q}(\boldsymbol{\theta})^{-1}), \quad \boldsymbol{\theta} \sim \pi(\boldsymbol{\theta})$$

- $\mathbf{y} = \{y_1, \dots, y_n\}$  is a collection of observations of the response variable,
- $\mathbf{x} = (x_1, \dots, x_n)$  represents the Gaussian field,
- $\boldsymbol{\theta}$  is a vector of hyperparameters, i.e., the parameters that define the prior distributions,
- $\boldsymbol{\mu}(\boldsymbol{\theta})$  is the mean,
- $\mathbf{Q}(\boldsymbol{\theta})$  is the precision matrix (i.e., the inverse of the covariance matrix).

The aim is to model mobility over three categories - Workplaces ( $C_1$ ), Retail and Recreation ( $C_2$ ), and Transit Stations ( $C_3$ ) - across mainland Portugal using daily time data grouped per district.

$$Y_{jtc} = \beta_c + \text{temporal}_{jtc} + \text{lockdown}_{tc} + \text{stringency}_t + \text{temperature}_{jt} + u_j + v_j + \gamma_{t|c} + \phi_{t|c} + \delta_{jt|c}$$

where,

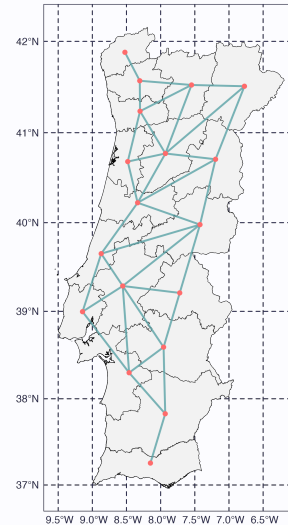
- $Y_{jtc}$  is the percentage change in mobility for district  $j$  ( $j = 1, \dots, 18$ ) on day  $t$  for category  $c$  ( $c = C_1, C_2, C_3$ ) and  $\beta_c$  represents a fixed effect per category.
- Random effects  $c$ ,

$$\underbrace{u_j + v_j}_{\text{spatial effects}} + \underbrace{\gamma_{t|c} + \phi_{t|c}}_{\text{temporal effects}} + \underbrace{\delta_{jt|c}}_{\text{space-time interaction}} \quad [10]$$

$$Y_{jtc} = \beta_c + \mathbf{temporal}_{jtc} + \mathbf{lockdown}_{tc} + \mathbf{stringency}_t + \mathbf{temperature}_{jt} + u_j + v_j + \gamma_{t|c} + \phi_{t|c} + \delta_{jtc}$$

- $\beta_c$  – category-specific fixed effects  $\beta_c = \sum_{i=1}^3 \beta_{C_i} \mathbb{I}(c = C_i)$ .
- $\mathbf{temporal}_{j,t,c}$  – category-specific linear trends, Fourier terms, day of the week and district holiday indicators.
- $\mathbf{lockdown}_{t,c}$  – full lockdown indicator with category-specific random effects.
- $\mathbf{stringency}_t$  – national movement stringency index [7]).
- $\mathbf{temperature}_{j,t}$  – daily mean temperature for district  $j$  on day  $t$ , with fixed and district-specific random effects.
- $u_j + v_j$  – spatial effects modeled using the Besag–York–Mollié (BYM) model [12, 11].
- $\gamma_{t,|,c}$  – correlated random temporal effect by mobility category  $c$ .
- $\phi_{t,|,c}$  – uncorrelated temporal effect by category  $c$ .
- $\delta_{j,t,|,c}$  – Type I space–time interaction, combining unstructured spatial and temporal effects.

- The dataset was split into a training set, **Feb 20, 2020 – Feb 20, 2022**, and a test set, **Feb 21, 2022 – Oct 15, 2022**, roughly a 75–25% train-test split.
- Models were estimated using the **R-INLA** package [13].
- Model fit was evaluated with **DIC** and **WAIC**, while predictive performance was assessed using **MAE** and **RMSE** on both training and test sets [10, 14].



**Figure 6:** Geographical representation of mainland Portugal's districts with overlaid adjacency graph.

## Results

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**Table 1:** Model specification and fit statistics

Model	DIC	WAIC	MSE (Train)	RMSE (Train)	MSE (Test)	RMSE (Test)
<b>model</b>	260647.71	278280.91	5.89	9.21	19.98	27.35

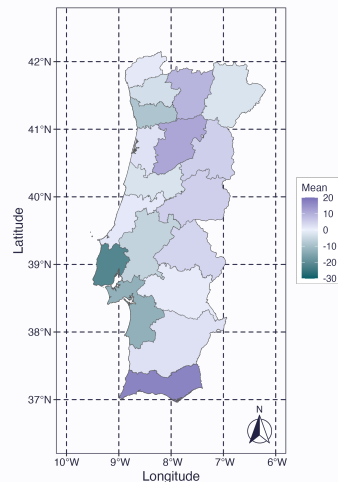
**Table 2:** Posterior estimates for the model, mean, standard deviation (SD), quantiles at 2.5%, 50% and 97.5%, and mode.

Covariate	Coefficient	Mean	SD	Q 2.5%	Q 50%	Q 97.5%	Mode
<b>category</b>	$\beta_{C_1}$	-15.76	5.46	-26.56	-15.73	-5.14	-15.73
<b>category</b>	$\beta_{C_2}$	22.38	5.39	11.84	22.36	33.03	22.36
<b>category</b>	$\beta_{C_3}$	15.52	5.28	5.26	15.49	25.97	15.49
<b>temporal features</b>	$\alpha$	...	...	...	...	...	...
<b>dow [Saturday]</b>	$\eta_{d_2}$	-0.30	1.19	-2.64	-0.30	2.05	-0.30
<b>dow [Sunday]</b>	$\eta_{d_3}$	-0.34	2.22	-4.71	-0.34	4.02	-0.34
<b>national holiday</b>	$\theta_1$	-29.47	1.25	-31.92	-29.47	-27.01	-29.47
<b>regional holiday</b>	$\theta_2$	-6.78	1.35	-9.44	-6.78	-4.13	-6.78
<b>commemorative holiday</b>	$\theta_3$	-12.99	2.01	-16.94	-12.99	-9.05	-12.99
<b>lockdown</b>	$\psi$	-12.02	2.77	-17.49	-12.02	-6.56	-12.02
<b>stringency</b>	$\zeta$	-0.77	0.03	-0.83	-0.77	-0.72	-0.77
<b>temperature</b>	$\omega$	0.68	0.17	0.35	0.68	1.02	0.68

# Results

**Table 3:** Posterior estimates for the model hyperparameters, posterior mean, posterior standard deviation (SD), quantiles at 2.5%, 50% and 97.5%, and Mode.

Covariate	Parameter	Mean	SD	Q 2.5%	Q 50%	Q 97.5%	Mode
<b>fixed effects</b>	$\tau$	0.01	0.00	0.01	0.01	0.01	0.01
<b>dow</b>	$\tau_{\eta}$	0.02	0.00	0.01	0.02	0.02	0.02
<b>lockdown</b>	$\tau_{\psi}$	0.06	0.02	0.02	0.06	0.12	0.06
<b>temperature</b>	$\tau_{\omega}$	2.33	0.52	1.48	2.27	3.53	2.16
spatial comp.	$\tau_u$	283.67	139.87	69.91	261.17	596.34	200.66
unstructured comp.	$\tau_v$	0.01	0.00	0.01	0.01	0.02	0.01
AR time effect (AR1)	$\tau_{\gamma}$	0.01	0.00	0.01	0.01	0.01	0.01
correlation term (AR1)	$\rho$	0.98	0.00	0.98	0.99	0.99	0.99
time effect	$\tau_{\phi}$	0.02	0.00	0.02	0.02	0.02	0.02
space-time interaction	$\tau_{\delta}$	2.06	3.13	0.38	1.16	9.61	0.54

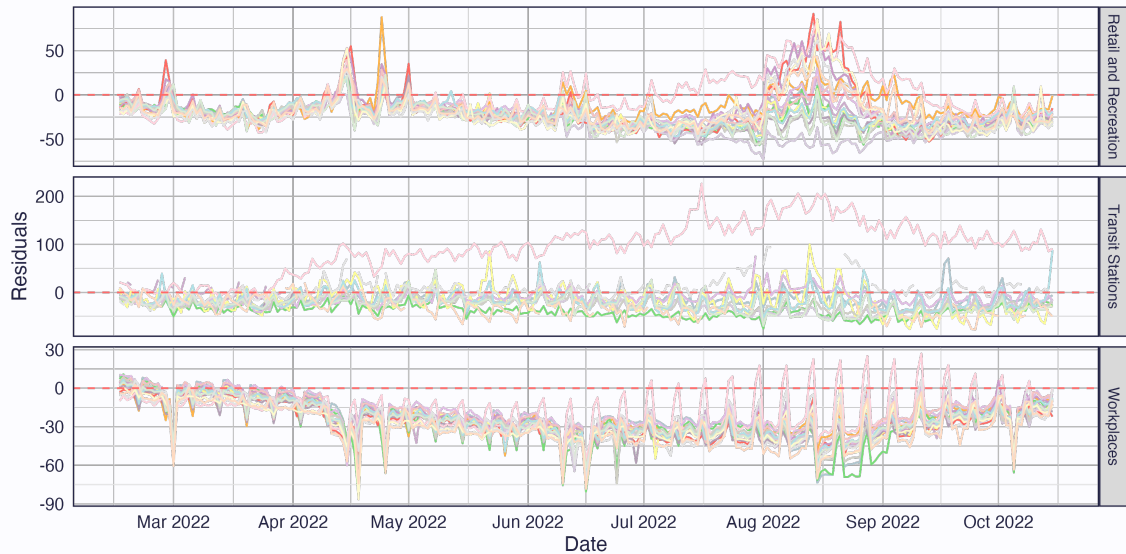


**Figure 7:** Posterior mean of the structured spatial main effect  $u_j + v_j$ .

# Results



# Results



# Results

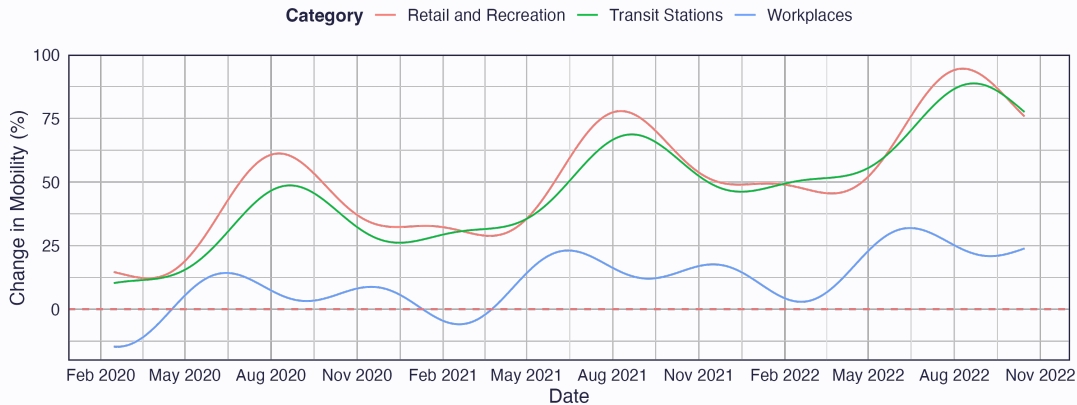


Figure 8: Time series of the underlying mobility seasonal patterns, baseline  $B_{C_i}$ , linear and Fourier terms, per category.

## Discussion

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- **Fourier terms** and **linear trends** significantly capture the **seasonal patterns** of each mobility category.
- **Holiday indicators** are highly significant, especially in the **Workplace category**, for capturing mobility disruptions.
- **Stringency** and **lockdown measures** jointly **reduce mobility**, with lockdowns amplifying the effects of stricter restrictions; however, the **stringency index alone** is **insufficient to capture abrupt changes**.
- Entering a **full lockdown** leads to an average **12% reduction** in mobility across all categories.
- **Temperature** has a **small but significant positive effect**, increasing mobility by approximately **0.68% per degree rise**.

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# Thank you for your attention

Any questions?

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